CASCADES FROM DESTRUCTION OF FLUID RESERVOIRS

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The hydrodynamic problem is examined of how a fluid cascade, which is formed during the destruction of a reservoir, interacts with a protective barrier designed to withstand a rolling wave of fluid. A method is given to determine the height of the protective barrier as a function of the reservoir parameters and the distance to its lateral surface.

Cascades can occur when a fluid reservoir is partially or totally destroyed. Protective structures in the form of earthen embankments, which are used in domestic practice, are designed for quasi-static containment of the escaping fluid; therefore, they cannot contain a dynamic cascade. Protective barriers can be used in order to increase the safety of reservoir fields near industrial sites and roads. The problem is that such barriers must withstand rolling waves of fluid formed when the resrvoir is destroyed. The height of each barrier must depend both on the reservoir height, and on the distance to it. A theory is presented to calculate the desired height, a method is given to solve the corresponding mathematical problems, the problems are solved, and the results are analyzed.

1. Basic Equations. We examine a layer of fluid of depth h(x, y, t) moving along a plane at an angle α to the horizontal. The motion of the fluid is characterized by the depth-averaged velocity components u_i, with i = 1, 2, along the Ox- and Oy-axes. The fluid is assumed to be incompressible, therefore the continuity equations, integrated over the layer depth, are reduced to an equation relating this depth to the averaged components of the flow velocity:

$$\frac{\partial h}{\partial t}+\frac{\partial h u_k}{\partial x_k}=0. \tag{1}$$

A repeated index k indicates summation from 1 to 2.

The basic factors, which determine the development of the fluid cascade, are the force of gravity and the inertia of the fluid. The viscosity and other rheological properties appear only at the final stage of the flow process and play practically no role in this problem. Therefore, friction can be neglected, and the forces of motion are taken as the horizontal components of the hydrostatic pressure gradient, which is formed by the variation in the layer depth. Under these assumptions, we have

$$\frac{\partial u_i}{\partial t}+u_k\frac{\partial u_i}{\partial x_k}+g\frac{\partial h}{\partial x_i}=-g\cos\alpha_i. \tag{2}$$

In the one-dimensional case, which models the propagation of the fluid cascade when the walls of a flat channel or reservoir break, the defining Eqs. (1) and (2) are simplified:

$$\frac{\partial x^\nu h}{\partial t}+\frac{\partial x^\nu hu}{\partial x}=0,\quad \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g\frac{\partial h}{\partial x}=-g\cos\alpha. \tag{3}$$

Here the superscript ν is either 0 or 1 for plane or cylindrical symmetry.

2. Problem Formulation. We now examine the following problem: An infinitely long channel, filled with fluid to a constant depth H_0, lies between two walls at x = ±R. At time t = 0 the walls are destroyed instantaneously and the heretofore stagnant fluid flows away on both sides. Protective barriers are placed at distances (L − R) to the right and left of the channel walls, that is, at the points x = ±L. The protective barriers are designed to prevent fluid from penetrating into the region |x| > L. The question is asked, how high must the protective barriers be, so that the fluid cascade cannot move over the barriers?

I. M. Gubkin Institute of Oil and Gas, Moscow. Translated from Inzhenerno-fizicheskii Zhurnal, Vol. 61, No. 5, pp. 726-731, November, 1991. Original article submitted November 30, 1990.

This question can be answered by solving the following boundary problem:

$$\frac{\partial h}{\partial t}+\frac{\partial hu}{\partial x}=0,\quad \frac{\partial hu}{\partial t}+\frac{\partial}{\partial x}\left(hu^2+\frac{gh^2}{2}\right)=0;$$
$$h(x,\ 0)=\begin{cases}H_0 & \text{for } 0\leqslant x\leqslant R,\\ 0 & \text{for } R<x\leqslant L;\end{cases}\quad u(x,\ 0)=0,\ u(0,\ t)=u(L,\ t)=0. \tag{4}$$

Here the height H_b of the protective barriers is found as the maximum height of the fluid at points $x = \pm L$.

The other problem, for a cylindrical reservoir, is formulated in an analogous manner. A cylindrical reservoir of radius R and height H_0, initially filled with fluid, is instantaneously destroyed. Here a wave flows out to a protective barrier at a distance (L − R) from the edge of the reservoir. Here we must find the barrier height H_b, which is defined as the maximum of the function h(L, t). The solution to this problem is found from the following formulation:

$$\frac{\partial h}{\partial t}+\frac{1}{r}\frac{\partial rhu}{\partial r}=0;\quad \frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+g\frac{\partial h}{\partial r}=0;$$
$$h(r,\ 0)=H_0 \text{ for } 0\leqslant r\leqslant R;\ h(r,\ 0)=0 \text{ for } R<r\leqslant L; \tag{5}$$
$$u(r,\ 0)=0;\ u(0,\ t)=u(L,\ t)=0.$$

3. Method of Solution. These problems are solved with the aid of Godunov's method [1, 2] and the use of the solutions to the problem of the destruction of an arbitrary discontinuity in the system of quasi-linear hyperbolic equations (1) and (2). If these equations are integrated over the area of a cell $x_j \le x \le x_{j+1}$ and $t_{m-1} \le t \le t_m$ ($\Delta x = x_{j+1} - x_j$ and $\Delta t = t_m - t_{m-1}$), then the values of $h_{j+1/2}$ and $u_{j+1/2}$, averaged over the interval (x_j, x_{j+1}), are obtained from the system of recurrence relations

$$h_{j+1/2}^{m}=h_{j+1/2}^{m-1}+[hu]_j^{j+1}\frac{\Delta t}{\Delta x};$$
$$h_{j+1/2}^{m}u_{j+1/2}^{m}=h_{j+1/2}^{m-1}u_{j+1/2}^{m-1}+[hu^2+gh^2/2]_j^{j+1}\frac{\Delta t}{\Delta x}, \tag{6}$$

in which the superscript indicates the time, and the symbol [ ] indicates the difference of the enclosed quantity at the nodes indicated by the upper and lower indices.

The system (6) could be used to calculate the desired functions at the time t_m from their known values at time t_{m-1}; however, the system includes unknown parameter values h and u at the cell boundaries. According to Godunov, these values are taken from the solution to the problem of the decay of an arbitrary discontinuity in the system of Eqs. (1) and (2). This discontinuity arises from the interaction of two flows with constant but different values $h_{j-1/2}$ and $u_{j-1/2}$ to the left of the contact boundary and $h_{j+1/2}$ and $u_{j+1/2}$ to the right. This problem is self-similar, and its solution at $x = x_j$ gives the desired values of h_j and u_j on the boundary of each cell.

4. Decay of an Arbitrary Discontinuity. Gladyshev [3] examined this problem in its general form for open waterways. We show that in the case of a rectangular channel, this problem has 41 different solutions, depending on the ratio of the parameters to the right and the left of the separation boundary. Because of the self-similarity, these solutions consist of simple fast (R^+) and slow (R^-) waves which satisfy the equations

$$R^+:\xi=u+\sqrt{gh},\ u-2\sqrt{gh}=\text{const};$$
$$R^-:\xi=u-\sqrt{gh},\ u+2\sqrt{gh}=\text{const},$$

where $\xi = x/t$ is the self-similarity variable. The problem solution also contains shock fast (S^+) and slow (S^-) waves (hydraulic discontinuities), states for which (h and u) satisfy the equation

$$(u-u_0)^2=\frac{g}{2h_0}(h-h_0)^2\left(1+\frac{h_0}{h}\right),$$

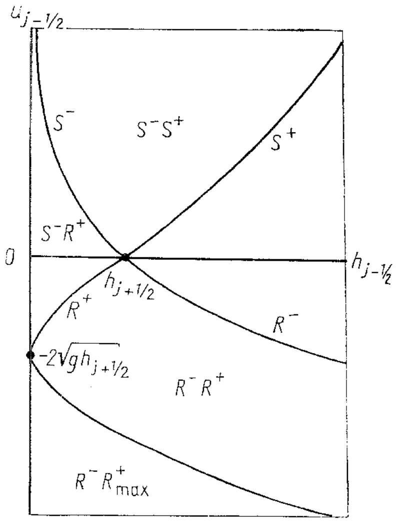

Fig. 1. Diagram for the decay of an arbitrary discontinuity.

where h_0 and u_0 are the flow parameters ahead of the shock wave. The difference between fast and slow shock waves is in their propagation velocities, which is determined by progression conditions [4].

Figure 1 shows the plane of the variables $h_{j-1/2}$ and $u_{j-1/2}$, broken into regions for fixed values of $h_{j+1/2}$ and $u_{j+1/2}$. In each region the solution is the same for h and u. (Because of Galileo's principle, there is no loss of generality if we take $u_{j+1/2} = 0$.) The solution to the problem is composed of various combinations of fast waves (S^+ and R^+) and slow waves (S^- and R^-), which follow each other, and are separated by different regions of constant parameters. The wave combinations which comprise the solution in each region are shown in Fig. 1. Here $R_{max}{}^+$ represents the fast simple wave of maximum intensity.

The order of solving the problem is as follows. We start with the given parameters $h_{j-1/2}^{m-1}$, $u_{j-1/2}^{m-1}$ and $h_{j+1/2}^{m-1}$, $u_{j+1/2}^{m-1}$ from the (m − 1)-th time slice and determine the combination of waves which correspond to the solution for the decay of an arbitrary discontinuity. Then we compute the value of the self-similarity variable ξ, which gives the position of each of the waves. The values of h_j and u_j, which correspond to $\xi = -u_{j+1/2}^{m-1}$, determine the value of the required parameters on the boundary of the (j − 1)-th and j-th cells. In an analogous manner, the required variables h_{j+1} and u_{j+1} on the boundaries of the j-th and (j + 1)-th cells are computed from the values of $h_{j+1/2}^{m-1}$, $u_{j+1/2}^{m-1}$ and $h_{j+3/2}^{m-1}$, $u_{j+3/2}^{m-1}$. Finally, Eqs. (6) are used to compute the values of the hydrodynamic parameters $h_{j+1/2}{}^m$ and $u_{j+1/2}{}^m$ for the following time slice.

5. Results of the Calculations. Figure 2 shows the results for the instantaneous destruction of a cylindrical reservoir of radius 10 m and height H_0 = 10 m. The reservoir is surrounded by a protective barrier of radius L = 20 m. The figure shows curves of typical shapes of the rolling fluid wave at sequential moments of time. The basic stages of the process can be followed clearly: a tongue of fluid propagates towards the barrier and the fluid level in the reservoir decreases; the cascade impacts the protective barrier and throws fluid sharply upwards along it; and a reverse wave of fluid, reflected from the barrier, propagates towards the center of the reservoir, which by now has a deep depression. The maximum rise of the liquid H_b at the barrier is 5.3 m in this case. Namely, at this height the protective barrier prevents fluid from penetrating beyond it.

The shape of the rolling fluid wave has an analogous form for all other cases, both planar and radial.

During the calculations, we varied the only dimensionless parameter L/R, which defines the location of the barrier with respect to the reservoir. Figure 3 shows the dependence of the dimensionless barrier height H_b/H_0 sufficient to contain the moving fluid behind the barrier as a function of the dimensionless distance L/R. From the curves it can be seen that for a plane of symmetry (ν = 0), the protective barrier should be much higher than for the case of cylindrical symmetry (ν = 1). Thus, for example, if the barrier is at one radius distance from the edge of the reservoir (L/R = 2), its height should be 1.01 in the planar

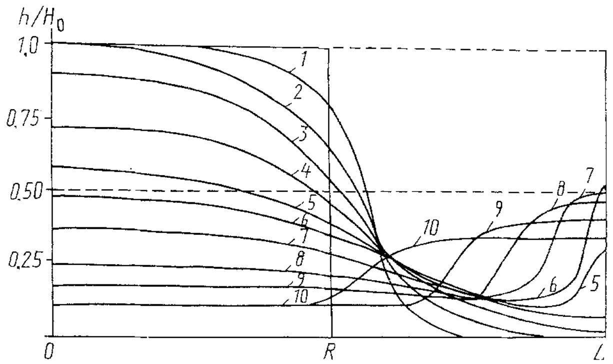

Fig. 2. Profile of the fluid cascade for L/R = 2 and $\alpha = 0$: curves 1-4 are constructed with an interval $\Delta T = 0.45$ sec; curves 5-10 with an interval of $\Delta t = 0.9$ sec.

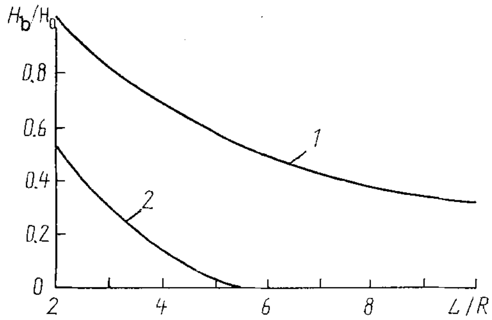

Fig. 3. Protective barrier height vs. the distance from the edge of the reservoir: 1) $\nu = 0$; 2) $\nu = 1$.

case, but only 0.53 in the cylindrical case. If the distance from the barrier to the edge of the reservoir is three reservoir radii (L/R = 4), then $H_b/H_0 = 0.64$ in the planar case but 0.15 in the cylindrical case. Finally, for cylindrical reservoirs, a barrier at a distance of 5 times the radius from the edge of the reservoir is almost not needed at all, and an earthen embankment is completely sufficient. At the same time, the height of a protective barrier should never be less than half the height of the reservoir in the plane case.

Other factors which affect the fluid motion were also varied during the calculations. Friction was considered by including hydraulic losses on the right side of Eq. (2) [5]. However, it turned out that the results hardly changed, even in the case of significant resistance. As it was assumed a priori, the gravitational forces are much larger than the friction forces.

We also analyzed the effect of the cascade propagating at an angle α to the horizontal. Figure 4 shows the profiles of a cascade at various times for the case when the region between the reservoir and the barrier has a rise defined by $\operatorname{tg} \alpha = 0.2$. While the shape of the cascade depends on α, it turned out that the barrier height was practically unchanged in this case, so the results are valid for both a rise ($\alpha > 0$) and a drop ($\alpha < 0$).

The computational algorithm described above also was used to solve the two-dimensional problem of (1) and (2), which models the partial destruction of the reservoir. The problem is posed as follows: At time t = 0, a vertical opening with the same height as the reservoir and of width b is formed in the wall of the reservoir. The fluid, which has been stationary until this time, starts to flow through the opening, such that its rolling wave moves towards a protective barrier, established at some distance from the edge of the reservoir. Here it is assumed that the fluid level in the reservoir drops as the fluid flows out. It turns out that the computational algorithm based on Godunov's method is also valid in this case.

The results are as follows. If the cascade flows through a vertical plane of symmetry, then the resultant profile is analogous to that in Fig. 2; however, the curves of the propagating fluid cascade are completely different than in the one-dimensional case: the fluid is observed to spread in a direction perpendicular to the plane of symmetry. The height of

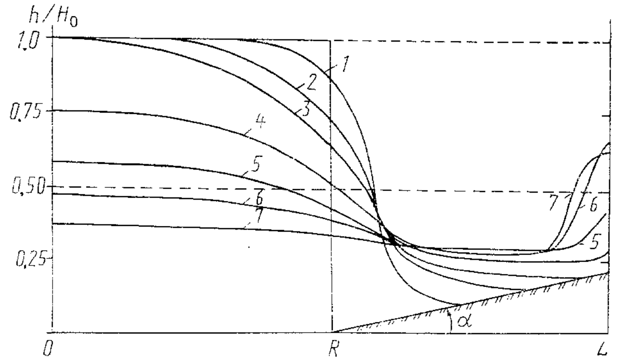

Fig. 4. Profile of a fluid cascade for L/R = 2 and tg α = 0.2; curves 1-7 are constructed for a time interval Δt = 0.45 sec.

the fluid rise when it impacts the protective barrier is less than in the axisymmetric case. Thus, for example, for the case analogous to that shown in Fig. 2 (H_0 = 10 m, L − R = 10 m, and b = 2 m), the maximum fluid rise is 2.1 m instead of 5.3 m for the axisymmetric case. However, this number is much larger than would be expected from existing predictions.

Conclusions. Results of the investigations show that in practically all cases the height of the protective barrier should be much higher than computed earlier, and the currently used earthen embankments for reservoirs near industrial sites and roads cannot protect them from the danger that arises when the reservoir breaks.

NOTATION

h, depth of the fluid layer; u_k, velocity component of the fluid rolling wave; g, acceleration due to gravity; α_i, angle formed by the velocity vector and the direction of gravity; H_0, initial height of the fluid layer in the reservoir; L, distance of the protective barrier from the center of the reservoir; H_b, barrier height; b, width of the opening for partial destruction of the reservoir; R^+ and R^-, fast and slow simple waves; S^+ and S^-, fast and slow shock waves (hydraulic jumps); ξ, self-similarity coordinate; x, spatial coordinate; t, time; m and j, calculational cell numbers with respect to time and space.

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